Theory of Distortion
"Natural Theory of Relativity"
by
Fayaz Tahir

The fact that light takes a definite amount of time to travel from one point to another has not been considered by researchers but rather the fact that the speed of light is constant in free space is considered. In the theory, that I am going to set forth, I will consider the above fact that light takes a definite amount of time to travel from one point to another point in space along with the fact that the speed of light is a constant in free space, meaning that it has the same speed in all directions.

1. Postulates of Natural Relativity

1. The speed of light is the speed of our information.

2. Measurements, in a frame of reference, of physical quantities of another frame of reference in relative motion taken with the speed of our information as universal constant and requiring definite amount of time to travel between two relative frames, are distorted.

2. Simple Thought Experiment "Time Dilation"

Let us run a simple thought experiment. In order to determine the time dilation of a clock at one point in space observed by an observer in a relative inertial or locally relative inertial frame moving with a constant velocity "v", I will actually put two clocks in the frame at rest relative to the moving frame, with a spatial distance "x" apart. Then, I will calculate the time the moving observer measures of the two clocks according to his measurability. The two equations describing the measurement of time of two different but synchronized clocks in the rest frame will be scrutinized in the limiting case when the spatial distance "x" between the two clocks at rest tends to zero, hence making the two different times merge into one observation equal in magnitude. So, while the moving observer moves away from one clock it is simultaneously moving closer to the second one.

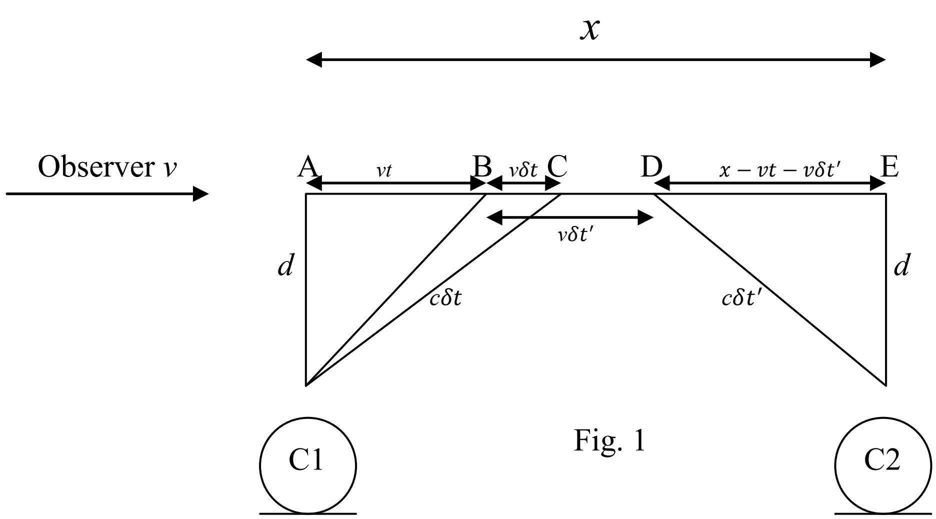

Fig. 1

In the above Figure 1, an observer is moving from the left to the right. Two clocks C1 & C2 synchronized with each other are placed *"x"* distance apart. When the observer approaches right on top of the clock at a height of *"d,"* the clocks C1 &C2 start synchronically. After an elapse of time *"t,"* the observer reaches point B in its moving frame. Both clocks at rest measure time t as both are synchronized with each other in the rest frame. Now, the observer tries to get the information of the time of C1, and by the time *"δt"* the information, at the speed of light (hereafter considered to be the speed of our information), reaches him he has reached point C. Information speed takes definite amount of time (time light or light-like signals take to reach from C1 to observer at point C). The observer at point "C" records this time to be *"t_1"* according to the equation below:

$$(vt + v\delta t)^2 + d^2 = (c\delta t)^2 \qquad 1$$

Where $\delta t = t_1 - t$ & c is the speed of our information

δt is the definite amount of time in which the information reaches from C1 to observer at point C. In the same manner, we can observe the fact that when the same observer gets the information of the time of C2, he has reached the point "D". "D" does not necessarily need to be on the right of point "C", it can be on the left of C depending upon the prevailing conditions between the observer and the clocks. The same observer records the time of "C2" according to the same power of measurability with which he measured the time of "C1." He measures it *"t_2"* given by equation 2 below:

$$(x - vt - v\delta t')^2 + d^2 = (c\delta t')^2 \qquad 2$$

Where $\delta t' = t_2 - t$

$\delta t'$ is the definite amount of time in which the information reaches from C2 to observer at point "D." Substituting the value of *"d^2"* from equation 1 into 2, we get the following:

$$(x - vt_2)^2 + (c\delta t)^2 - (vt_1)^2 = (c\delta t')^2$$
$$x^2 - 2xvt_2 + (c^2 - v^2)(t_1^2 - t_2^2) - 2c^2 t(t_{1-}t_2) = 0$$

Now in the limiting case when *"x"* tends to zero t_1 & t_2 merge to become T as follows:

$$T = \frac{t}{(1-v^2/c^2)} \qquad 3$$

The time dilation T is independent of both *"d"* and the observer going either to the left or to the right above the clock. This is the equation for time dilation. This equation does not refute the equation for time dilation as given by Lorentz Transformations. In Lorentz Transformations [Beiser], we see how light (electromagnetic wave phenomenon) [Adams & Smith] warps space and time to keep its speed constant for all observers and hence constant in all directions.

3. Analysis of the Time Dilation

There is no absolute frame of reference because our speed of information is finite and takes a definite amount of time to travel from one point to another in space. Due to the lack of an absolute frame of reference, time becomes proper time in a certain frame, because its measurement made by different observers in different relative inertial or locally relative inertial frames moving with relative velocity becomes distorted with respect to their own frames of references. This relative distortion in the measurement of that particular proper time arises due to two parameters *"v"* & *"c"* competing each other in equation 3 of my "Theory of Distortion /Natural Theory of Relativity," hereafter, called TD/NTR. Now, let us analyze the equation for time dilation given by TD/NTR as follows for relative velocities less than *"c"*:

$$T = \frac{t}{(1-v^2/c^2)}$$

$$T = t(1 - \frac{v^2}{c^2})^{-1}$$

$$T = t(1 + \frac{v^2}{c^2} + \frac{v^4}{c^4} + \frac{v^6}{c^6} + \frac{v^8}{c^8} + \cdots \infty) \quad 6$$

$$T = t + \ t(\frac{v^2}{c^2} + \frac{v^4}{c^4} + \frac{v^6}{c^6} + \frac{v^8}{c^8} + \cdots \infty) \quad 7$$

The most important term in the binomial series expansion on the right hand side of equation 6 is the first term of "1." This represents the factor for the proper time and it remarkably retains the reality of the relative inertial or locally relative inertial frame in the scenario when there is no relative velocity. We can observe that the only thing responsible for distortion in our actual value of proper time *"t"* in equation 7 above is the infinite series as below:

$$\frac{v^2}{c^2} + \frac{v^4}{c^4} + \frac{v^6}{c^6} + \frac{v^8}{c^8} + \cdots \infty$$

Infinite Series = Distortion

So, we can say that proper time and other physical quantities as well, are distorted by relative motion in combination with the characteristics of light, in the form of infinite series. Distortion is a relative effect measured in a relative frame and has nothing to do with the physical reality that lies within the frame of reference under investigation. “One frame measures the other distortedly in relative motion.” Basically when distortion, i.e., infinite series, is got rid-off, then we are left with the actual picture of the frame of reference as follows:-

$$T = t$$

Distortion, as far as I think, is produced due to human incapability to measure with infinite speed of information in “relative motion.”

TD/NTR calculates the actual value of bending of a ray of light near the Sun. For relative velocities far less than the speed of our information, the distorted values given by TD/NTR are pretty close to the observed experimental values, as in the case of the recession of the perihelion of Mercury, since the theory is capable enough to approximate to the first order the infinite series. TD/NTR also gives the actual value of the recession of the perihelion of Mercury, which will come later.

4. On the Immeasurability

According to TD/NTR, immeasurability, construed as human limited extent of measurability, is one of the causes of distortion. Immeasurability is due to the fact that our speed of information is finite and it takes definite amount of time to reach from one point in one frame of reference to another point in another frame of reference. We can never receive information instantaneously without the intervention of time from one point in space to another. Immeasurability together with relative motion causes distortion.

The three things responsible for Distortion are as follows:

1. Relative speed between two frames of references.
2. Constancy of the speed of light in all directions.
3. Finite speed of light.

The above three causes have woven the fabric of our distortion in the form of infinite series.

$$\text{Distortion} = \text{Immeasurability} + \text{Relative motion} = \text{Infinite series}$$

5. Kinetic Energy of a Material Object "Distorted v/s Actual"

Let us try to extract the physical reality from the distortion of the kinetic energy of a body of matter moving with constant velocity *"v"*. Since the theory that I have developed incorporates the relative inertial frame or locally relative inertial frame, therefore everything will be considered in view of relative inertial frames of references unless specified otherwise. We will take for granted the expression for the relative momentum to be calculated in the same way as derived [Beiser] by researchers but here with a slight modification in view of TD/NTR. Hence, the expression for the relativistic mass that conserves momentum is as follows:

$$m(v) = \frac{m_o}{(1-v^2/c^2)} \quad \text{distorted} \qquad 8$$

$$m(v) = m_o \quad \text{actual} \qquad 9$$

If we multiply equation 8 with "c^2" as was done remarkably by researchers, we get

$$m(v)c^2 = \frac{m_o}{(1-v^2/c^2)}c^2 \qquad 10$$

$$m(v)c^2 = m_o c^2(1-v^2/c^2)^{-1}$$

$$m(v)c^2 = m_o c^2\left(1+\frac{v^2}{c^2}+\frac{v^4}{c^4}+\frac{v^6}{c^6}+\frac{v^8}{c^8}+\infty\right)$$

$$m(v)c^2 = m_o c^2 + m_o v^2 + m_o c^2\left(\frac{v^4}{c^4}+\frac{v^6}{c^6}+\frac{v^8}{c^8}+\infty\right) \qquad 11$$

$$m(v)c^2 = m_o c^2 + m_o v^2 + \text{infinite series (distortion)}$$

Total relative inertial energy (Total Energy) is distorted by the infinite series, so after getting rid-off distortion, we get

$$m(v)c^2 = m_o c^2 + m_o v^2 \qquad 12$$

$$E = m_o c^2 + m_o v^2 \qquad 13$$

$$E = m_o c^2(1+v^2/c^2) \qquad 14$$

We know that energies are of two types either kinetic or potential. Conjecturing [Beiser] in line with the same notion we can conclude that equation 13 is more likely to be the expression for the total relative inertial energy of a body i.e., rest mass energy [Beiser] and relative inertial kinetic energy. The Equation 11 is the distorted picture of physical reality in relative motion, but equation 12 is the picture of the true nature of our frame of reference produced by TD/NTR. I believe, in equation 14 lies the information connecting the remarkable concept of relative mass with gravitational or inertial mass.

Rewriting equation 14 as

$$E = m_o' c^2$$

$$m_o' = m_o(1 + \frac{v^2}{c^2})$$

$$m_{relative} = m_{gravitational}\left(1 + \frac{v^2}{c^2}\right) = m_{inertial}\left(1 + \frac{v^2}{c^2}\right) \qquad 15$$

However, the values are pretty close at Newtonian speeds.

Let me explain equation 15 now. There occurs absolutely no change in mass, as discussed before, m_g remains equal to m_i. Instead what happens is that the relative inertial kinetic energy of a body of matter is stored in the body as apparent mass.

$$\text{Relative Inertial Kinetic Energy} = m_{g\ or\ i}v^2 = \Delta m_{apparent\ increase\ in\ m_{g\ or\ i}} c^2$$

$$\Delta m_{apparent\ increase\ in\ m_{g\ or\ i}} = m_{g\ or\ i}\frac{v^2}{c^2} \qquad 15^*$$

$$m_{g\ or\ i} + \Delta m_{apparent\ increase\ in\ m_{g\ or\ i}} = m_{g\ or\ i} + m_{g\ or\ i}\frac{v^2}{c^2} = m_{g\ or\ i}(1 + \frac{v^2}{c^2}) = m_r$$

$$m_r = m_{g\ or\ i} + \Delta m_{apparent\ increase\ in\ m_{g\ or\ i}} \qquad 15^{**}$$

$$m_{relative} = m_{gravitational}\left(1 + \frac{v^2}{c^2}\right) = m_{inertial}\left(1 + \frac{v^2}{c^2}\right) \approx m_{locally\ rel.\ inertial\ (planetary\ motion)}$$

It should be kept in mind that the above equation 15 is not the binomial approximation of equation 8 at Newtonian speeds. Furthermore, planetary motion is locally inertial because the orbital velocity of a planet does not change appreciably over a small interval of time. Hence, the linear orbital acceleration of a planet is very small as compare to ordinary Newtonian accelerations of ordinary objects on earth.

Immeasurability, together with relative motion, gives rise to distortion in measurement of the total relative inertial energy. Since the velocity of an object constitutes a moving frame of reference, therefore, when we measure its physical quantities, we get a distorted picture, due to relative motion, which has absolutely nothing to do with reality. Now, we can measure the actual picture of physical quantities like energy, time and mass. Equation 13 is an example of the actual value of the total relative inertial energy of a body in relative inertial frame moving at relative velocity *"v."*

If our laboratory starts moving with a relative speed, comparable to the speed of our information, with which we are getting information, then it starts distorting the information we are getting. Hence, we measure the physical quantities of other frames distortedly.

Now, let us try to extract the relative inertial kinetic energy of a body, moving with zero linear acceleration, constituting an inertial frame of reference. Using the conventional method [Beiser], we have

$$KE = \int F ds$$
$$= \int \frac{dp}{dt} ds$$
$$= \int \frac{ds}{dt} dp$$
$$= \int v dp$$

Now $p = m(v) * v = \frac{m_o}{(1 - v^2/c^2)} * v$

$$KE \quad = \int v dp$$

Using Integration by parts, we have

$$KE = vp - \int p dv + C$$

$$KE = \frac{m_o v^2}{(1 - v^2/c^2)} + C$$

In the case of inertial motion with constant velocity, $dv = 0$, therefore, the integral on the right above is zero. To calculate the value of the constant of Integration *"C,"* we see that the relative inertial kinetic energy is zero at velocity equal to zero. Therefore, the constant of Integration is also zero.

We obtain the results

$$KE = \frac{m_o v^2}{(1 - v^2/c^2)}$$

Now, expanding the terms of the expression on the right above with binomial series, we see

$$KE = m_o v^2 (1 + \frac{v^2}{c^2} + \frac{v^4}{c^4} + \frac{v^6}{c^6} + \cdots \infty)$$

$$KE = m_o v^2 + m_o v^2 (\frac{v^2}{c^2} + \frac{v^4}{c^4} + \frac{v^6}{c^6} + \cdots \infty) \qquad 16$$

$$KE = m_o v^2 + \text{ Infinite series} \qquad 17$$

$$KE = m_o v^2 \text{ (Actual)} \qquad 18$$

We see above that the relative inertial kinetic energy according to TD/NTR is again distorted by an infinite series.

Newtonian non-inertial kinetic energy formula $\frac{m_o v^2}{2}$ is for comparatively large linear accelerations starting from zero velocity, where as the relative inertial kinetic energy formula KE = mv^2 is for constant velocity and zero linear acceleration. Since the planets have comparatively small linear accelerations, therefore, TD/NTR's relative inertial kinetic energy formula is a better candidate for appreciably small linear accelerations as that of the planets revolving around the Sun than the Newtonian one.

$$E = m_o c^2 + m_o v^2 \qquad 19$$

When we compare the second term (relative inertial kinetic energy) on the right hand side of the above equation 19 with equation 18, we see that it's the same and hence in agreement.

Let us analyze the picture of momentum [Beiser] as follows:

$$p = m(v) * v = \frac{m_o}{(1 - v^2/c^2)} * v$$

Now once again in line with TD/NTR, the momentum of matter is given by, ignoring the terms of distortion, as follows

$$p = m_o v$$

valid for all velocities less than *"c."* It also predicts the momentum of photon equal to "$p = mc$". Photon has relative as well as gravitational/inertial mass ($m_{relative} = 2m_{gravitational\ or\ inertial}$ for photon only). Its prediction can be seen when we calculate the bending of ray of light near the Sun by using TD/NTR.

Equation 13 can also be written as

$$E - pv = m_o c^2 \qquad 20$$

Where $p = m_o v$

6. Force and Acceleration of a Material Body "Distorted v/s Actual"

For the calculation of force we will use the same conventional method [Beiser] as we used in the calculation of kinetic energy.

$$p = m(v) * v = \frac{m_o}{(1-v^2/c^2)} * v$$

$$F = \frac{dp}{dt} = \frac{d}{dt}(\frac{m_o v}{1 - v^2/c^2})$$

$$F = m_o \frac{d}{dt}(\frac{v}{1 - v^2/c^2})$$

$$F = m_o \frac{d}{dv}(\frac{v}{1 - v^2/c^2}) \frac{dv}{dt}$$

$$F = m_o a \frac{d}{dv}(\frac{v}{1 - v^2/c^2})$$

We arrive at the following equation

$$F = m_o a \frac{(1+\frac{v^2}{c^2})}{(1-\frac{v^2}{c^2})^2} \qquad 22$$

Let us analyze this equation in the light of TD/NTR as follows:

$$F = m_o a \left(1 + \frac{v^2}{c^2}\right)(1 - \frac{v^2}{c^2})^{-2}$$

$$F = m_o a \left(1 + \frac{v^2}{c^2}\right)(1+2\frac{v^2}{c^2} +3\frac{v^4}{c^4} + 4\frac{v^6}{c^6} + \ \dots \infty)$$

$$F = m_o a \left(1 + \frac{v^2}{c^2}\right) + m_o a \left(1 + \frac{v^2}{c^2}\right)(2\frac{v^2}{c^2} +3\frac{v^4}{c^4} + 4\frac{v^6}{c^6} + \ \dots \infty) \qquad 23$$

$$F = m_o a \left(1 + \frac{v^2}{c^2}\right) + \text{Infinite series (distortion)}$$

Getting rid of the distortion terms as before we see that

$$F = m_o a \left(1 + \frac{v^2}{c^2}\right) \qquad 24$$

Rewriting equation 24 above as

$$F = m_o' a$$

Where

$$m_o' = m_o(1 + \frac{v^2}{c^2})$$

$$m_{relative} = m_{gravitational}\left(1+\frac{v^2}{c^2}\right) = m_{inertial}\left(1+\frac{v^2}{c^2}\right) \qquad 25$$

Or in short

$$m_r = m_g\left(1+\frac{v^2}{c^2}\right) = m_i\left(1+\frac{v^2}{c^2}\right) \qquad 26$$

We see that the slightly modified form of Newton's second law of motion as given by equation 24 approximates Newton's second law at Newtonian velocities as follows:-

$$F = m_r a \approx m_g a = m_i a \qquad v <<< c \qquad 27$$

7. Test of TD/NTR

1. Bending of Ray of Light near a Massive Body like the Sun "Actual"

Newton's second law of motion, as slightly modified by TD/NTR, is given by

$$F = m_r a = m_g a\left(1+\frac{v^2}{c^2}\right) = m_i a\left(1+\frac{v^2}{c^2}\right) \qquad 28$$

Equating Newton's Universal Force of Gravitation between two bodies with the slightly modified Newton's force of relative inertial motion as given by equation 28 above, we see

$$\frac{GM_s m_g}{r^2} = m_r a = m_g\left(1+\frac{v^2}{c^2}\right)a = m_i(1+\frac{v^2}{c^2})a$$

$m_g = m_i$, so a can be replaced by g in the gravitational portion of the above expression.

Setting $r = R, a = g$ and $v = c$ for a photon passing near the sun in the above equation, we get

$$\frac{GM_s m_g}{R^2} = 2m_g g$$

$$GM_s = 2gR^2 \qquad 29$$

Putting this value of GM_s from equation 29 into equation 3 of appendix for "*k,*" we get equation 1 of appendix in the slightly modified form as

$$c^2 = 2(1+\epsilon)gR$$

$$\epsilon = \frac{c^2}{2gR} - 1 \approx \frac{c^2}{2gR} \qquad \epsilon \gg 1 \qquad 30$$

Putting this value of eccentricity ϵ from equation 30 above in equation 14 of appendix

We get

$$\delta = \frac{2}{\epsilon} = \frac{4gR}{c^2} \text{ radians} \qquad 31$$

Putting the following data in equation 31 above, gives the bending of ray of light when it passes near the sun to be

Radius of Sun = 695,500,000 m
Acceleration due to gravity of Sun = 29.74g=29.74*9.81 m/s^2
Speed of light = 299792458 m/s

$$\delta = 1.86 \quad \text{arcsecond}$$

The above value is in pretty agreement with General Relativity!

When we solve equation 29 to get the mass of the sun, we see that it gives double the value of the mass of the sun as contrary to the Newtonian value. This is due to the fact that photon creates relative motion between itself and the Sun. Both the bodies involved that is the sun and the photon feel the same apparent effect as long as relative motion exists (in the form of relative mass) of each other's mass (gravitational or inertial) in relative motion.

According to TD/NTR when the photon passes near the sun, its centripetal acceleration $a = \frac{c^2}{R} \ggg\ggg g$ for unbounded orbit, remains constant but its relative mass (which is always double its gravitational or inertial mass) converts the gravitational mass of the sun to relative mass (which becomes equal to double its gravitational mass in the case of interaction with the photon only) due to relative effects. Therefore, the Universal force of gravitation becomes double in the case of the Sun and Photon interaction, due to relative inertial motion, which eventually deflects the photon by double the amount as given by Newton's Classical physics.

Let us analyze the equation below

$$F = \frac{GM_s m_g}{r^2} = m_r a = m_{g\ or\ i}(1 + \frac{v^2}{c^2})a$$

Substituting the value of $m_{g\ or\ i} * \frac{v^2}{c^2} = \Delta m_{apparent\ increase\ in\ m_{g\ or\ i}}$ from equation15*, we have

$$F = \frac{GM_s m_g}{r^2} = m_{g\ or\ i}a + m_{g\ or\ i}\frac{v^2}{c^2}a = m_{g\ or\ i}a + \Delta m_{apparent\ increase\ in\ m_{g\ or\ i}}a \qquad 31^*$$

In other words, as from above, the apparent increase in gravitational or inertial mass of photon and eventually of sun is the cause of double the force of gravity between the sun and the photon at photon speed $v = c$, when it passes near the Sun.

2. Recession of the Perihelion of Mercury
"Distorted v/s Actual"

As I said earlier in Article 3 "Analysis of the Time dilation" last paragraph, that at velocities far less than the speed of information, TD/NTR gives the distorted values which are in pretty close agreement with the experimental values. Let us check this by calculating the distorted value of the recession of the perihelion of Mercury.

Rewriting equation 23, and approximating it to the first order we get

$$F = m_o a\left(1+\frac{v^2}{c^2}\right) + m_o a\left(1+\frac{v^2}{c^2}\right)(2\frac{v^2}{c^2} + 3\frac{v^4}{c^4} + 4\frac{v^6}{c^6} + \ \ldots\infty)$$

$$F \approx m_o a\left(1+\frac{v^2}{c^2}\right) + m_o a * 2\frac{v^2}{c^2} \approx m_o a\left(1+3\frac{v^2}{c^2}\right)$$

$$F \approx m_o a\left(1+3\frac{v^2}{c^2}\right) \qquad 32$$

Equating Newton's Universal Force of Gravitation between two bodies with the distorted relative inertial Newton's force as given by equation 32 above, we see

$$\frac{GM_s m_g}{r^2} \approx m_r a = m_g(1+3\frac{v^2}{c^2})a$$

$$\frac{GM_s}{r^2} \approx (1+3\frac{v^2}{c^2})a$$

Approximating to the first order by using binomial series expansion, we have

$$a = \frac{GM_s}{r^2\left(1+3\frac{v^2}{c^2}\right)} \approx \frac{GM_s\left(1-3\frac{v^2}{c^2}\right)}{r^2} \qquad 33$$

Inserting the above value of centripetal acceleration, because $m_i = m_g$ hence a = centripetal acceleration, instead of Newtonian GM/r^2 in to the equations of Article 3 [www] "On the Planetary motion" of Appendix, we get

$$\frac{d^2u}{d\theta^2} + u = \frac{GM}{h^2}(1 - 3\frac{v^2}{c^2}) \qquad 34$$

We know that $h = v*r$ = constant, $v = h/r$ & $u=1/r$, putting this value of v in to the above equation we get

$$\frac{d^2u}{d\theta^2} + u = \frac{GM}{h^2}(1 - 3\frac{v^2}{c^2})$$

$$= \frac{GM}{h^2} - 3\frac{GM}{c^2}u^2$$

$$3\frac{GM}{c^2}u^2 + u + \frac{d^2u}{d\theta^2} - \frac{GM}{h^2} = 0$$

$$P = 3\frac{GM}{c^2} \qquad Q = \frac{GM}{h^2}$$

$$Pu^2 + u + \left(\frac{d^2u}{d\theta^2} - Q\right) = 0$$

Solving this algebraically for u gives

$$u = \frac{-1 + \sqrt{1 - 4P\left(\frac{d^2u}{d\theta^2} - Q\right)}}{2P}$$

The second term inside the square root is much less than 1, so we can closely approximate the solution using just the first couple of terms of the expansion, so we have

$$u = \frac{1}{2P}[-1 + \{1 - 4P\left(\frac{d^2u}{d\theta^2} - Q\right)\}^{1/2}]$$

$$u = \frac{1}{2P}[-1 + 1 - \frac{1}{2} * 4P\left(\frac{d^2u}{d\theta^2} - Q\right) + \frac{{}^1\!/_2({}^1\!/_2 - 1)}{2!}\{-4P\left(\frac{d^2u}{d\theta^2} - Q\right)\}^2 + ...]$$

Simplifying and re-arranging terms, we get

$$(1\text{-}2PQ)\frac{d^2u}{d\theta^2} + u = Q(1 - PQ) - P(\frac{d^2u}{d\theta^2})^2$$

The last term on the right hand side of the above equation is negligibly small, so we essentially have an equation of the form of equation 18 in Article 3 of Appendix, with

$$(1\text{-}2PQ)\frac{d^2u}{d\theta^2} + u = Q(1 - PQ)$$

$$\Omega = \frac{1}{\sqrt{1-2PQ}} \approx \frac{1}{1-PQ} \qquad P = \frac{1}{Q(1-PQ)}$$

Hence the distorted (the actual is with the value of P reduced by three times as will become clear later) Newton-Kepler orbit is described by the relationship below

$$r(\theta) = \frac{\frac{1}{Q(1-PQ)}}{1 + k\cos(\frac{\theta}{1-PQ})}$$

This again is the equation of an ellipse, except that the period of the radial function is not exactly equal to the period of the angular position θ. The angular travel necessary to go from one perigee to the next, for example, is not 2π, but rather $2\pi(1\text{-}PQ)$. Hence, the ellipse recesses by the amount $2\pi PQ$ radians per revolution. Putting the values of P & Q from above, we have

$$2\pi PQ = 2\pi * 3\frac{GM}{c^2} * \frac{GM}{h^2} = 6\pi\left(\frac{GM}{hc}\right)^2 \text{ radians per revolution}$$

This gives the recession in units of radians per revolution of the Planet. To convert this to units of arc-seconds per century for the planet Mercury, noting that Mercury completes 414.9 revolutions per century, we multiply the above expression by

$$(414.9)\frac{360}{2\pi}3600$$

So, the distorted (measured by astronomical instruments) value for the recession of the planet Mercury, due to relative motion between the Sun and Mercury only, in units of arc-seconds per century, is

$$6\pi\left(\frac{GM}{hc}\right)^2 *(414.9)\frac{360}{2\pi}3600$$

By substituting the data [Red aka] below, we get

$h = v_{min}r_{max}$ (at Apogee)

$v_{min} = 38{,}860\ m/s$
r_{max} =69,816,927000 m
$c = 299{,}792{,}458\ m/s$
$G = 6.67 *\ 10^{-11}\ N.m^2/kg^2$
$M = 1.99 *\ 10^{30}\ kg$

$$6\pi\left(\frac{GM}{hc}\right)^2 *(414.9)\frac{360}{2\pi}3600 = 42.96 \approx 43\ \text{arc} - \text{seconds per century}$$

Once again another value by TD/NTR is in pretty good agreement with General Relativity!

Now, let us attempt to find the actual value for the recession of the planet Mercury, due to relative motion between the sun and Mercury only.

The actual equation of Newton's Law of motion in relative motion after getting rid of distortion caused by the characteristics of the speed of our information in combination with relative motion between the Sun and Mercury, as slightly modified by TD/NTR, is given by equation 24 above as

$$F = m_o a\left(1 + \frac{v^2}{c^2}\right)$$

Comparing the above equation with Newton's Universal Gravitational Force, we have

$$\frac{GM_s m_g}{r^2} = m_g(1 + \frac{v^2}{c^2})a$$

Approximating to the first order by using binomial series expansion, we have

$$a = \frac{GM_s}{r^2\left(1 + \frac{v^2}{c^2}\right)} \approx \frac{GM_s\left(1 - \frac{v^2}{c^2}\right)}{r^2}$$

Inserting the above value of centripetal acceleration, instead of the one given by equation 33 above, in to the equations of Article 3 "On the Planetary motion" of Appendix, we get this time a slightly different equation

$$\frac{d^2u}{d\theta^2} + u = \frac{GM}{h^2}(1 - \frac{v^2}{c^2})$$

$$\frac{GM}{c^2}u^2 + u + \frac{d^2u}{d\theta^2} - \frac{GM}{h^2} = 0$$

The above equation shows that the only change now is that the value of P is reduced by three times. The rest of the calculations are as before. So, we see that this time the recession is reduced by three times. Therefore, the actual recession of Mercury, which is observed by astronomical instruments on earth distortedly to be 42.96 arc-seconds per century, just due to the relative motion between Mercury and the Sun, is given by

$$\frac{1}{3} *6\pi \left(\frac{GM}{hc}\right)^2 *(414.9)\frac{360}{2\pi}3600 = 14.32 \quad \text{arc} - \text{seconds per century}$$

$$F = \frac{GM_s m_g}{r^2} = m_r a = m_{g\ or\ i}(1 + \frac{v^2}{c^2})a$$

Considering the above equation, let me give the explanation for the recession of the perihelion of Mercury. The acceleration in this case is given by $a = g(r) = \frac{v^2}{r}$ for bounded orbit of Mercury around the Sun. Inserting this value of acceleration in the above equation, we have

$$F = \frac{GM_s m_g}{r^2} = m_r a = m_g(1 + \frac{v^2}{c^2})\ \frac{v^2}{r}$$

When Mercury leaves the perihelion point in its orbit around the Sun, its orbital speed starts decreasing, but very slowly, with almost negligible linear acceleration, as compare to appreciable large Newtonian accelerations in the case of ordinary objects on earth. Simultaneously, its radial distance from the Sun starts increasing, so the overall effect reduces the gravitational force of the Sun on Mercury as given by the above expression. This overall effect i.e., the difference of gravitational force between two points in the orbit, is more pronounced in Natural Orbits of TD/NTR than it is in Kepler-Newton orbits because of the addition of an extremely small extra term in the above expression for force. Thus, it gives a relative deviation from the Kepler-Newton orbits in the form of recession. This relative decrease in the pull of the Sun on Mercury causes the Mercury to have a recession at aphelion in its orbit. Similarly, when the planet leaves the recessed aphelion point, its orbital speed starts increasing. Simultaneously, its radial distance starts decreasing, producing an overall effect which increases the gravitational force of the Sun on Mercury as given again by the same above expression. This relative increase in the pull of the Sun on Mercury causes the planet to have a recession at perihelion of the same amount that it had at the aphelion, and that's how, I believe, the cycle continues. Newton's Universal Force of Gravitation, the pull of the Sun on Mercury, fluctuates by the same amount as given before by equation 31*

$$\Delta m_{apparent\ change/fluctuation\ in\ m_{g\ or\ i}} a$$

3. Gravitational Red Shift

As we know that light is an electromagnetic wave phenomenon [Adams & Griffiths] and has wave characteristics as well. From Plank's Law, we know that the energy associated with the wave of a photon (electromagnetic radiation) is [Beiser] $E = h\boldsymbol{\nu} = m_{g\ or\ i}c^2$. Inserting this in equation 14 above, we see

$$E = m_o\, c^2(1{+}v^2/c^2)$$

$$E = h\boldsymbol{\nu}(1{+}v^2/c^2) = h\boldsymbol{\nu}_{relative}$$

$$\boldsymbol{\nu}_{relative} = \boldsymbol{\nu}(1{+}v^2/c^2)$$

Where h is the Plank's Constant and $\boldsymbol{\nu}$ is the frequency of electromagnetic radiation and v is the relative velocity between the observer and the source of radiation. When a body falls in the gravitational field of matter in motion, it loses potential energy and gain relative inertial kinetic energy. By equating the said two energies we get

$$m_{g\ or\ i}v^2 = m_{g\ or\ i}gh \quad \Rightarrow \quad v^2 = gh$$

Inserting this expression for $v,^2$ in the above equation, for gravitational shift in frequency, we have

$$\boldsymbol{\nu}_{relative} = \boldsymbol{\nu}(1{+}\frac{gh}{c^2})$$

The effect is very small but measurable on Earth using the Mössbauer effect and was first observed in the Pound-Rebka experiment [Beiser].

APPENDIX

1. On the Newton's Cannon

According to Newton's Theory [Adams] a body of matter thrown with a velocity tangential to the surface of the earth, the point of projection being at the earth, equal to $v = \sqrt{2gR}$ where g is the acceleration due to gravity of the earth at the point of projection and R is the radius of earth, will escape the earth's gravity by making a parabolic path. The range of velocities greater than

$v = \sqrt{gR}$ and less than $v = \sqrt{2gR}$ will force the object to remain in bounded elliptical orbit around the earth. At a horizontal tangential velocity greater than $v = \sqrt{2gR}$ the orbit will become hyperbolic. We need an infinite amount of horizontal escape velocity to project a body of matter in an exactly horizontal path along a tangential line (eccentricity of the straight-line orbit = ∞) at the surface of projection. The governing equation, from classical Newtonian mechanics, for all types of horizontal velocities is the following:

$$V_h = \sqrt{(1+\epsilon)gR} \qquad 1$$

ϵ is the eccentricity of the orbit that will be adopted by matter. The values g & R are as mentioned below.

Let us prove equation 1 from the well known polar equation [Adams] of an orbit (like the orbit of a planet around the Sun) that is

$$r = \frac{h^2}{k(1+\epsilon * cos\theta)} \qquad 2$$

Where h is the angular momentum per unit mass of planet around the bigger body and

$$k = GM = gr^2 \qquad 3$$

Where G = Constant of Universal Gravitation

M = Mass of the bigger body
g = Acceleration due to gravity (as a function of distance) experienced by the revolving planet
r^2 = Radial distance between the center of mass of the two bodies

The above equation can be reduced to

$$r = \frac{h^2}{k(1+\epsilon)} \qquad 4$$

at the perihelion (the point on the orbit which is closest to the Sun) if we set the value of arbitrary angle $\theta = 0^\circ$.

Now $h = v * r$ is the angular momentum per unit mass of the smaller revolving body

Putting this value of *h* along with the value of *k* from equation 3 above, we eventually arrive at

$$V = \sqrt{(1+\epsilon)gr}$$

$$V = \sqrt{(1+\epsilon)gR} \qquad r = R \text{ at the surface of body and g also at the surface}$$

$$v = \sqrt{gR} \qquad \epsilon = 0;\ \text{circular orbit}$$

$$\sqrt{gR} < v < \sqrt{2gR} \qquad 0 < \epsilon < 1; \text{elliptical orbit}$$

$$v = \sqrt{2gR} \qquad \epsilon = 1; \text{parabolic orbit}$$

$$v > \sqrt{2gR} \qquad \epsilon > 1; \text{hyperbolic orbit}$$

2. On the Eccentricity of Hyperbolic Orbit

Let us do some coordinate geometry to find the eccentricity [Dakin, Thomas & Stewart] in terms of the angle that the two asymptotes (on the right and left of $y -$ axis) of the hyperbola make with each other. In order to do so we need to find the slope of the asymptote at infinity. Let point A be the point on the y-axis A$(0, a)$, Point P on the Hyperbola as P(x, y) and point B$(x, -a)$ on the horizontal line $y = -a$. Let PB be always the perpendicular distance of the point P from the Horizontal line $y = -a$ since it's a condition of eccentricity of conic sections. Point A is fixed and points P and B are varying their positions.

Forming the ratio of two lengths $\frac{AP}{PB} = \epsilon = \frac{\sqrt{(y-a)^2+(x-0)^2}}{\sqrt{(x-x)^2+(y+a)^2}}$ 5

Rearranging terms, we obtain

$$y^2(1-\epsilon^2) + x^2 - 2ay(1+\epsilon^2) + a^2(1-\epsilon^2) = 0 \qquad 6$$

After completing squares, equation 6 can also be written as

$$y(1-\epsilon^2) = \pm\sqrt{4a^2\epsilon^2 - x^2(1-\epsilon^2)} + \frac{a^2(1+\epsilon^2)^2}{(1-\epsilon^2)} \qquad 7$$

If we subtract the term $-a(1+\epsilon^2)$ from equation 7 above we get

$$y(1-\epsilon^2) - a(1+\epsilon^2) = \pm\sqrt{4a^2\epsilon^2 - x^2(1-\epsilon^2)} + \frac{a^2(1+\epsilon^2)^2 - a(1-\epsilon^4)}{(1-\epsilon^2)} \qquad 8$$

Differentiating equation 6 w.r.t. x and rearranging terms, we get

$$\frac{dy}{dx} = \frac{-x}{y(1-\epsilon^2) - a(1+\epsilon^2)} \qquad 9$$

Substituting from equation 8 into equation 9, we get

$$\frac{dy}{dx} = \frac{-x}{\pm\sqrt{4a^2\epsilon^2 - x^2(1-\epsilon^2)} + \frac{a^2(1+\epsilon^2)^2 - a(1-\epsilon^4)}{(1-\epsilon^2)}} \qquad 10$$

$$\lim_{x\to\infty} \frac{dy}{dx} = \frac{\pm 1}{\sqrt{(\epsilon^2 - 1)}} \qquad 11$$

In equation 11 above, the slope of the right asymptote to the hyperbola at infinity is

$$\frac{1}{\sqrt{(\epsilon^2 - 1)}} \qquad 12$$

If we denote the angle that the asymptote of the hyperbola makes with the x-axis as α, then the tangent of this angle α is equal to the slope of asymptote at infinity

$$\tan\alpha = \frac{1}{\sqrt{(\epsilon^2 - 1)}} \approx \frac{1}{\epsilon} \qquad \epsilon \gg 1 \qquad 13$$

$$\tan\alpha \approx \alpha \qquad \epsilon \gg 1$$

$$\alpha \approx \frac{1}{\epsilon}$$

Now the angle "δ" that the asymptotes (one on the left and one on the right) make with each other is double the angle α that is $\delta = 2\alpha$. This is the deviation from the matter-free straight path of Newton's first Law of Motion.

$$\delta = \frac{2}{\epsilon} \qquad 14$$

3. On the Planetary Motion

For a planet revolving around the Sun [www] at a distance r, the Newtonian equations of motion state that the planet has a centripetal acceleration of magnitude GM/r^2 in the direction of the Sun. Since the planet is confined to a single plane, so its position as a function of time can be expressed in terms of the radial magnitude *r(t)* and an angular position *θ(t)* as

$$x(t) = r(t)\cos\theta(t) \qquad y(t) = r(t)\sin\theta(t)$$

The second derivatives of these coordinates are

$$\ddot{x} = (\ddot{r} - r\dot{\theta}^2)\cos\theta - (r\ddot{\theta} + 2\dot{r}\dot{\theta})\sin\theta = -\frac{GM}{r^2}\cos\theta$$
$$\ddot{y} = (\ddot{r} - r\dot{\theta}^2)\sin\theta + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\cos\theta = -\frac{GM}{r^2}\sin\theta$$

Since the absolute value of θ is arbitrary, these equations are equivalent to the conditions obtained by setting the value of the angle equal to 0

$$\ddot{r} - r\dot{\theta}^2 = -\frac{GM}{r^2} \qquad\qquad r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \qquad\qquad 15$$

Multiplying the equation on the right above by r, we get

$$r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} = \frac{d}{dt}(r^2\dot{\theta}) = 0$$

and therefore the quantity in parentheses is constant, i.e.,

$$r^2\dot{\theta} = h$$

This represents the conservation of angular momentum, and it applies to the central force law. The constancy of this quantity also accounts for Kepler's second law, because the incremental area swept out by the position vector in an incremental time is $dA = (1/2)r^2 d\theta$. Making the substitution $d\theta/dt = h/r^2$ into the left hand equation 1 gives

$$\ddot{r} - \frac{h^2}{r^3} = -\frac{GM}{r^2} \qquad\qquad 16$$

Notice that for a circular orbit all the derivatives of r vanish, and this equation reduces to $h^2 = rGM$. Making the substitution $h = r^2\omega$ where $\omega = d\theta/dt$, we have $\omega^2 r^3 = GM$, in accordance with Kepler's third law.

We can also use the relation $d\theta/dt = h/r^2$ to express the derivative of r with respect to time in terms of the derivatives of r with respect to the angular position θ. We have

$$\dot{r} = \left(\frac{d\theta}{dt}\frac{dt}{d\theta}\right)\frac{dr}{dt} = \frac{h}{r^2}\frac{dr}{d\theta}$$

$$\ddot{r} = h\left(\frac{d\theta}{dt}\frac{dt}{d\theta}\right)\frac{d}{dt}\left(r^{-2}\frac{dr}{d\theta}\right) = \frac{h^2}{r^2}\frac{d}{d\theta}(r^{-2}\frac{dr}{d\theta})$$

Inserting this expression for the second derivative of r into equation 16 and simplifying gives

$$\frac{d}{d\theta}\left(\frac{1}{r^2}\frac{dr}{d\theta}\right) - \frac{1}{r} = -\frac{GM}{h^2}$$

Notice that the quantity in parentheses is just the negative of the derivative of $1/r$ with respect to θ. Therefore, letting $u = 1/r$, we have the simple harmonic equation

$$\frac{d^2u}{d\theta^2} + u = \frac{GM}{h^2} \qquad\qquad 17$$

In general the solution of an equation of the form

$$\frac{1}{\Omega^2}\frac{d^2u}{d\theta^2} + u = \frac{1}{p} \qquad 18$$

for constants Ω and p can be written in the form

$$u(\theta) = \frac{1}{p}(1 + k\cos(\Omega\theta))$$

where k is a constant of integration. In the present case we have $\Omega = 1$ and $p = h^2/GM$. Recalling that $r = 1/u$, the path of the planet in the gravitational field of the Sun of mass M is

$$r(\theta) = \frac{p}{1 + k\cos(\Omega\theta)} = \frac{h^2/GM}{1 + k\cos\theta}$$

If the magnitude of k is less than 1, this is the polar equation of an ellipse with the origin at one focus (Kepler's first law), and with semi-latus rectum $p = h^2/GM$.

References:

- Dakin, R. I. Porter, *Elementary Analysis* (1971)
- Thomas, G. B. Jr., Finney, R. L., *Calculus and Analytic Geometry* (Jan 1988)
- Griffiths, D. J., *Introduction to Electrodynamics*, 1st Edition
- Stewart, J., *Calculus* (1999), 4th Edition, Brooks/Cole Publishing Company, an ITP Company
- Adams, R. A., *Calculus: A Complete Course* (2002)
- *www.mathpages.com/home/kmath280/kmath280.htm*
- Beiser, A, *Concepts of Modern Physics* (2003)
- Giancoli, D. C., *Physics for Scientists and Engineers* 3rd Edition
- Abott, A. F., *Ordinary Level Physics* 2nd Edition
- Shutz F., Bernard, *A first course in General Relativity* 2nd Edition, Cambridge University Press
- Hawking, S., *A Stubbornly Persistent Illusion, Essential Scientific Works of Elbert Einstein* (2007), Running Book Publishers, Philadelphia
- Spiegel, M. R., *Vector Analysis with an Introduction to Tensor Analysis*, Schaum's Outline(2006)
- Red aka, *Our Nine Planets and their Specifications*, Oct 05 2007